

DESCRIPTION AND SOME ASPECTS OF CONTROL OF TEMPERATURE FLUCTUATIONS IN CATALYTIC WIRES AND GAUZES[†]

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Abstract—A model for the description and control of temperature fluctuations (flickering) in catalytic wires and gauzes is developed in this paper. The model is based on stochastic differential equations and experiments to test the model are proposed. A number of stochastic control problems for the control of flickering are formulated and two of the simplest ones (parameter optimizations) are solved.

1. THE PHENOMENON OF FLICKERING

Wires and gauzes that catalyze exothermic chemical reactions exhibit nonperiodic temperature fluctuations. These fluctuations can be of considerable magnitude [1-4]. The fluctuations, called flickering, appear to be a random phenomenon. The theoretical problem posed by flickering has not been solved satisfactorily. Outstanding problems include the need for a predictive or descriptive theory which describes the characteristics of the fluctuations and the need for a theory which leads to control of flickering. Since precious metal catalysts play an important role in many chemical processes [5, 6], the description and control of flickering is an important problem.

There are also practical reasons for studying flickering. In real operations, a precious metal catalyst is oxidized and lost during the course of the reaction. Assuming Arrhenius kinetics, the rate of loss of catalyst is proportional to e^{-k/T_w} , where k is a constant and T_w is the temperature of the wire. When the temperature of the wire fluctuates, T_w is a random variable. Let $\langle \cdot \rangle$ denote the ensemble average. Since

$$\langle e^{-k/T_w} \rangle \geq e^{-k/\langle T_w \rangle},$$

the rate of precious metal loss when the temperature fluctuates is greater than the rate of loss for constant temperature. An operating procedure which reduces flickering reduces the reactor operating cost.

Another operational reason for studying flickering is that the temperature fluctuations can lead to a cooling of the wire and premature extinction of the reaction. Extinction requires start up, which is a costly and time consuming procedure. Extinction can occur only if there are multiple steady states. To prevent multiple steady states, the temperature of the wire may be unacceptably high (high temperatures imply a greater loss of catalyst to side reactions), so that the engineer is forced to operate the reactor in a region with multiple steady states.

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Luss and co-workers studied flickering experimentally and theoretically [1-4]. They suggested two mechanisms for flickering. The first mechanism involves turbulent transport of the reactant. The second mechanism involves turbulent fluctuations at the wire/gas interface. In a real reactor, both processes occur, but their experiments and calculations indicate that the first mechanism may be more important than the second mechanism.

This paper introduces a model for the description of flickering, and some aspects of control are discussed. The model uses stochastic differential equations (SDE) and the diffusion approximation [7]. Experiments to test the SDE model are proposed and implications of the model for description and control of flickering are discussed.

The SDE model assumes that the reaction kinetics are deterministic, i.e., inherently predictable, and that flickering is caused by randomly driven turbulence. It is possible that the reaction kinetics are inherently chaotic [8, 9]. Hence, experiments to test the SDE model are important.

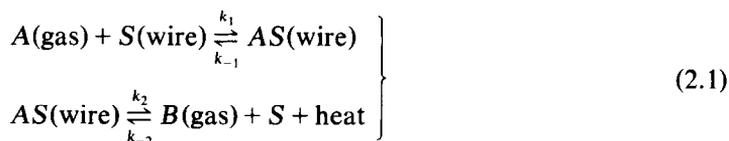
From a descriptive viewpoint, it may not be important if the SDE model or the chaotic model is "correct," since the SDE model can effectively describe chaotic dynamics. On the other hand, if the dynamics are chaotic, then control may not be a feasible goal.

This paper then has the following objectives: (1) To introduce a reasonable model for the description of flickering; this model is based on stochastic differential equations; (2) To suggest experiments to test the SDE model; (3) To set up the mathematical problems of the control of flickering; (4) To show how the diffusion approximation can be used to solve the simplest control problem (a parameter optimization).

In the next section, the chemical model and associated kinetic equations are introduced. The model is similar to the one used by Luss *et al.* [1-4], but different in that we use stochastic differential equations (SDE). In Sec. 3, the kinetic equations are non-dimensionalized and simplified. Experiments that could be used to test the SDE model are discussed in Sec. 4. In Sec. 5, we briefly discuss various aspects of the deterministic phase portrait as they apply to design and control. In Sec. 6, the control problem is formulated and in Sec. 7 it is shown how the diffusion approximation can be used to solve the control problem. Numerical results are presented in Sec. 8.

2. CHEMICAL MODEL AND KINETIC EQUATIONS

We consider the following very simple model (also used by Luss *et al.* [1-4]; but Schmitz and co-workers [8] have introduced more complicated models). The desired reaction is $A \rightarrow B$ and is assumed to satisfy the following kinetics



with the following assumptions:

- (1) Surface diffusion of AS and heat can be neglected[†];
- (2) Catalytic activity of the wire is uniform in time;
- (3) Only a monolayer of adsorbed species is formed;

[†]Recent work by D. S. Cohen and S. Rosenblat shows that flickering may be caused by the coupling of diffusion and reaction [21].

- (4) The temperature of the gas surrounding the wire, T_g , is constant in space and time and the reactant A is dilute in an essentially inert medium;
 (5) The back rate constants k_{-1} and k_{-2} can be ignored;
 (6) As a function of T_w ,

$$k_2 \propto e^{-E/RT_w};$$

(7) There are side reactions which cause loss of catalyst at a rate proportional to $e^{-E_1/RT_w} e^{-E_2/RT_g} S$.

These assumptions provide the simplest possible model for a catalytic wire. The mass action kinetic equations corresponding to (2.1) are

$$\left. \begin{aligned} \frac{d}{dt}[AS] &= k_1[A][S] - k_2[AS] \\ A\rho C_p \frac{dT_w}{dt} &= hP(T_g - T_w) + (-\Delta H)Pk_2[AS] \end{aligned} \right\} \quad (2.2)$$

In these equations, brackets denote concentration, A is the cross section of the wire, ρ is the density of the wire, C_p is the heat capacity of the wire, P is the perimeter of the wire, h is the heat transport coefficient, and ΔH is the heat of reaction.

The number of active surface sites is assumed to be constant, \bar{S} , so that

$$[AS] + [S] = \bar{S}. \quad (2.3)$$

Due to turbulent effects, the heat and mass transport coefficients and the reactant concentration fluctuate randomly [10]. In order to model this effect, we set

$$\left. \begin{aligned} k_1(t) &= k_{1c} + \tilde{k}_1(t) \\ h(t) &= h_c + \tilde{h}(t) \\ [A] &= [A]_c + [\tilde{A}(t)] \end{aligned} \right\} \quad (2.4)$$

In this equation k_{1c} , h_c , and $[A]_c$ are the average values of the mass transport coefficient, heat transport coefficient, and reactant concentration, and $\tilde{k}_1(t)$, $\tilde{h}(t)$, and $[\tilde{A}(t)]$ are zero mean random processes representing the effects of turbulence.

The fluctuations in the heat and mass transport coefficients are assumed to follow the turbulent velocity. If this velocity is denoted $u(t) = u_c + \tilde{u}(t)$, we assume the following [10]:

$$\frac{\tilde{k}_1(t)}{k_{1c}} = \frac{\tilde{h}(t)}{h_c} = \frac{\tilde{u}(t)}{u_c}. \quad (2.5)$$

The experiments of Luss and co-workers indicate that the intensity of turbulence, defined by

$$\epsilon_1 = \frac{\langle \tilde{u}(t)^2 \rangle^{1/2}}{u_c} \quad (2.6)$$

is between 0.05 and 0.11. The intensity of reactant fluctuations will be denoted by ϵ_2 , and is defined by

$$\epsilon_2 = \frac{\langle [\tilde{A}(t)]^2 \rangle^{1/2}}{[A]_c}. \quad (2.7)$$

3. DIMENSIONLESS KINETIC EQUATIONS AND DERIVATION OF THE SDE

In this section, Eqs. (2.1) are cast into a dimensionless form and the SDEs are derived. The scaling used here is different from the one used by Luss and co-workers [1-4]. The scaling chosen here is more appropriate for the control problem.

Let T_{gr} be a reference temperature (preferably cold) for the gas and define nondimensional variables τ , θ , Y , and β by

$$\begin{aligned}\tau &= k_{1c}[A]_c t \\ \theta &= [AS]/\bar{S} \\ Y &= T_w/T_{gr} \\ \beta &= T_g/T_{gr}.\end{aligned}\tag{3.1}$$

Equations (2.2) become

$$\frac{d\theta}{d\tau} = (1 - \theta) + (1 - \theta) \left[\frac{\bar{k}_1}{k_{1c}} + \frac{[\bar{A}]}{[A]_c} + \frac{\bar{k}_1}{k_{1c}} \frac{[\bar{A}]}{[A]_c} \right] - \frac{k_2\theta}{k_{1c}[A]_c}\tag{3.2}$$

and

$$\frac{A\rho C_p k_{1c}[A]_c}{k_c P} \frac{dY}{d\tau} = (\beta - Y) + \frac{\bar{h}}{h_c} (\beta - Y) + \frac{(-\Delta H)\bar{S}k_{1c}[A]_c}{h_c T_{gr}} \frac{k_2\theta}{k_{1c}[A]_c}.\tag{3.3}$$

Define parameters a , ω , α , and γ by

$$\left. \begin{aligned}a &= A\rho C_p k_{1c}[A]_c / h_c P \\ \omega &= (-\Delta H)\bar{S}k_{1c}[A]_c / h_c T_{gr} \\ \alpha e^{-\gamma Y} &= \frac{k_2}{k_{1c}[A]_c}\end{aligned}\right\}\tag{3.4}$$

A new time scale is defined so that $ad/d\tau = d/d\eta$. Then Equations (3.3, 4) become

$$\frac{1}{a} \frac{d\theta}{d\eta} = (1 - \theta) + \sqrt{\epsilon}(1 - \theta)\tilde{F}_1(t) - \alpha e^{-\gamma Y}\theta\tag{3.5}$$

$$\frac{dY}{d\eta} = (\beta - Y) + \omega\alpha e^{-\gamma Y}\theta + \sqrt{\epsilon}\tilde{F}_2(t)(\beta - Y).\tag{3.6}$$

In these equations, the random noise terms are defined so that

$$\begin{aligned}F_1(t) &= \frac{1}{\sqrt{\epsilon}} \left\{ \frac{\bar{k}_1}{k_{1c}} + \frac{[\bar{A}]}{[A]_c} + \frac{\bar{k}_1[\bar{A}]}{k_{1c}[A]_c} \right\} \\ F_2(t) &= \frac{1}{\sqrt{\epsilon}} \frac{\bar{h}(t)}{h_c}.\end{aligned}\tag{3.7}$$

Equations (3.5, 3.6) are the basic SDEs for modelling the temperature fluctuations in catalytic wires.

The parameter a is called the capacity of the wire; in industrial reactors it is very

large [4]. When a is large, the system (3.5, 3.6) can be reduced by the following procedure. Letting $a \rightarrow \infty$ in (3.5) and solving for θ_s , the steady state value, in the absence of fluctuations, gives

$$\theta_s = \frac{1}{1 + \alpha e^{-\gamma/Y}}. \tag{3.8}$$

This value is used in (3.6) to give

$$\frac{dY}{d\eta} = (\beta - Y) + \frac{\omega\alpha e^{-\gamma/Y}}{1 + \alpha e^{-\gamma/Y}} + (\beta - Y)\sqrt{\epsilon}\tilde{F}_2(t). \tag{3.9}$$

The physical interpretation of this result is that the surface concentration relaxes to perturbations on a time scale much faster than the temperature reacts. Consequently, turbulent transport effects do not appear in the SDE and only the turbulent heat transport coefficient appears.

The transformation from (3.5, 3.6) to (3.9) can be made more rigorous by a perturbation expansion in terms of the parameter a as $a \rightarrow \infty$ [7, 11]. This expansion will be discussed elsewhere. Equation (3.9) is the basic working equation for this paper.

Some typical values of the parameters are given in the sections on deterministic steady states and on numerical results. In the Appendix, the SDE model is compared with the model used by Luss and co-workers.

The random processes $\hat{F}_1(t)$ and $\tilde{F}_2(t)$ have zero means and the correlation functions of the processes can be obtained from (3.7). The correlation function of $\tilde{F}_2(t)$ only involves the heat transport coefficient, since

$$\langle \tilde{F}_2(t)\tilde{F}_2(t + \tau) \rangle = \frac{1}{\epsilon h_c^2} \langle \tilde{h}(t)\tilde{h}(t + \tau) \rangle. \tag{3.10}$$

The correlation function of $F_1(t)$ is a more complicated function, and involves a number of cross correlations.

4. AN EXPERIMENT TO TEST THE SIDE MODEL

Consider the high capacity case and rewrite (3.9) as a difference equation,

$$Y(\eta + \Delta\eta) - Y(\eta) = \Delta\eta\{\beta - Y(\eta) + \omega\alpha e^{-\gamma/Y(\eta)}/(1 + \alpha e^{-\gamma/Y(\eta)})\} + \Delta\eta(\beta - Y(\eta))\sqrt{\epsilon}\tilde{F}_2(t) + o(\Delta\eta). \tag{4.1}$$

Let $\Delta Y = Y(\eta + \Delta\eta) - Y(\eta)$. Equation (4.1) predicts that

$$\langle \Delta Y \mid Y(\eta) = y \rangle = \Delta\eta[(\beta - y) + \omega\alpha e^{-\gamma/y}/(1 + \alpha e^{-\gamma/y})] + o(\Delta\eta) \tag{4.2}$$

$$\langle (\Delta Y)^2 \mid y(\eta) = y \rangle = \Delta\eta(\beta - y)^2\epsilon + o(\Delta\eta), \tag{4.3}$$

where $\tilde{F}_2(t)$ is treated as being close to white noise (α legitimate assumption [3, 7]). Equation (4.3), in physical variables, reduces to

$$\langle (\Delta T_w)^2 \rangle \propto (T_g - T_w)^2. \tag{4.4}$$

By monitoring the temperature of the gas, the temperature of the wire, and the ensemble

average of fluctuations in the temperature of the wire, the relationship (4.4) can be tested; and the validity of the SDE model can then be examined.

5. SOME REMARKS ON THE DETERMINISTIC PHASE PORTRAITS

The deterministic phase portrait is obtained by studying the solutions of

$$f(Y, \beta) = \beta - Y + \frac{\omega\alpha e^{-\gamma Y}}{1 + \alpha e^{-\gamma Y}} = 0. \quad (5.1)$$

It is easy to show that (5.1) can have multiple solutions as β varies.

For the results presented in this paper, we use the following parameter values:

$$\left. \begin{aligned} T_{gr} &= 400^\circ\text{C} \\ 1 \leq \beta \leq 2.5 &\quad (\text{i.e., } 400^\circ\text{C} \leq T_g \leq 1000^\circ\text{C}) \\ \gamma &= 30 \\ \omega &= 2.5 \\ \alpha &= 10^4 \end{aligned} \right\} \quad (5.2)$$

The region of multiple steady states is determined by the simultaneous solution of

$$\left. \begin{aligned} f(Y, \beta) &= 0 \\ \frac{\partial f}{\partial Y}(Y, \beta) &= 0 \end{aligned} \right\} \quad (5.3)$$

A picture of the deterministic steady states is shown in Fig. 1. We find that multiple steady states exist for $1.43421 \leq \beta \leq 2.33451$. In this region, denote the three solutions of (5.1) by $y_0 \leq y_1 \leq y_2$; y_0 and y_2 are stable and y_1 is unstable.

As a function of the control parameter β , five cases are possible; they are shown in Fig. 1. They are as follows: (I) Only a high temperature steady state (which is stable) exists. This will be true if T_g is greater than the ignition temperature; i.e., $\beta > \beta_i$; (II) There is a high temperature stable steady state and a low temperature steady state of marginal stability at the ignition temperature $T_i \propto \beta_i$; (III) Two stable steady states (one of high temperature and one $\approx T_g$) coexist with an unstable steady state of moderate temperature. This is true when $T_e < T_g < T_i$, where T_e is the extinction temperature; (IV) A low temperature stable steady state and a high temperature steady state of marginal

Table 1. Typical values of parameters arising in the kinetic equations

Parameter	Value	Reference
T_g	20–1600°C	[2–4]
α	10^4	[2]
ω	$10^3/T_g$	[2]
γ	$11 \times 10^4/T_g$	[2]
a	$0.5-10^6$	[2–4]
ϵ	0.03–0.10	[2–4]

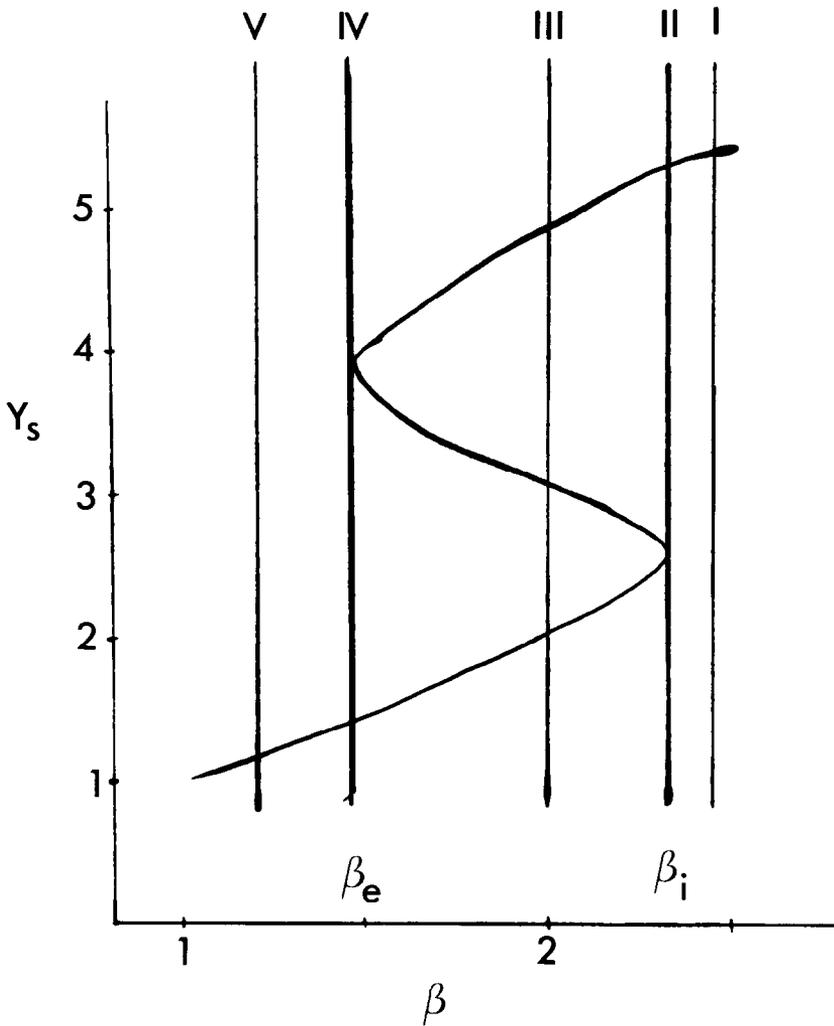


Fig. 1. Deterministic steady states for the catalytic wire [Eq. (5.1)].

stability coexist at the extinction temperature $T_e \propto \beta_e$; (V) A single stable low temperature steady state exists when $T < T_e$ ($\beta < \beta_e$).

The region of most interest in control parameter space is $\beta_i \leq \beta \leq \beta_e$. As $\beta \downarrow \beta_e$ or $\beta \uparrow \beta_i$, two steady states coalesce and annihilate each other; this behavior is called the marginal bifurcation [12] and should be contrasted with other critical dynamics in which three steady states coalesce.

In Refs. 12–15, a relatively complete theory for the description of fluctuations in systems with these types of bifurcations is given.

6. SOME STOCHASTIC CONTROL PROBLEMS IN CATALYTIC WIRES AND GAUZES

In this section, a number of control problems for the control of flickering in the catalytic wire are formulated. It is easy to formulate these problems, but not easy to solve them. In the next section, two of the simplest control problems are solved.

Imagine that at time $t = 0$, the reactor is started at some high temperature y_0 and is to run until $t = T$. Three types of control strategies are distinguished:

(1) Single stage open loop (also called parameter optimization) [16]. In this case, the value of β is fixed at $t = 0$ and kept at that value until $t = T$.

(2) Multiple stage open loop. In this case, the value of β is fixed over intervals $[t_k, t_{k+1}]$, with $t_0 = 0$, $t_N = T$.

(3) Closed loop/feedback control. In this case, the value of β is changed in $[0, T]$ according to the value of the temperature of the wire.

From an operational viewpoint, the single stage open loop control may be the most interesting, since it is the easiest to implement. From a mathematical viewpoint, the closed loop controls are the most interesting, since they provide the hardest problems.

We now discuss some possible objective functionals. Let y^+ be the maximum acceptable value of the temperature of the catalytic wire. Let $y^* = \frac{1}{2}(y_1(\beta) + y_2(\beta))$ be the midpoint between the unstable and high temperature steady states. Let $p(t, y, y_0; \beta)$ be the probability density function for $Y(t)$, given $Y(0) = y_0$. The first objective functional is the *containment* probability functional

$$J_1(T, \beta, y_0) = \int_0^T \int_{y^*}^{y^+} p(t, y, y_0; \beta) dy dt. \quad (6.1)$$

The maximum value of $J_1(T, \beta, y_0)$ is T and occurs if the system stays in $[y^*, y^+]$ for the entire operating time.

The next objective functional includes a penalty for loss of catalyst due to high temperatures. The rate of loss of catalyst is assumed to be

$$r = k_4 e^{-\gamma_2/\beta} e^{-\gamma_1/Y} S. \quad (6.2)$$

The deterministic fraction of catalyst remaining at time t is obtained by solving

$$\frac{dS}{dt} = -r, \quad S(0) = S_0, \quad (6.3)$$

so that

$$\frac{S(t)}{S_0} = \exp\left[-\int_0^t k_4 e^{-\gamma_2/\beta} e^{-\gamma_1/Y(s)} ds\right]; \quad (6.4)$$

and the expected fraction of catalyst remaining at time t , given $Y(t') = y_0$ is

$$f(t, t', y_0, \beta) = E_{y_0, t'}\left\{\exp\left[-\int_{t'}^t k_4 e^{-\gamma_2/\beta} e^{-\gamma_1/Y(s)} ds\right]\right\}. \quad (6.5)$$

In this equation, $E_{y_0, t'}$ is the expectation over all paths starting at $Y(t') = y_0$. As a second functional, set

$$J_2(T, \beta) = cJ_1(T, \beta, y_0) + (1 - c)f(T, 0, y_0, \beta) \quad (6.6)$$

with $0 \leq c \leq 1$ a weighting factor.

A third possible objective functional constrains the *rate* at which catalyst is lost and is

$$J_3(T, \beta, y_0) = cJ_1(T, \beta, y_0) - (1 - c)\frac{1}{T} \int_0^T E_{y_0, t'}\{k_4 e^{-\gamma_2/\beta} e^{-\gamma_1/Y} S(s)\} ds. \quad (6.7)$$

It is not hard to see that these optimization problems are quite difficult to solve; some progress can be made by using the diffusion approximation [17].

Associated with the SDE, (3.9), is a deterministic partial differential operator, L_y , defined by†

$$L_y = \frac{\epsilon}{2}(\beta - y)^2 \frac{\partial^2}{\partial y^2} + \left[(\beta - y) + \frac{\omega\alpha e^{-\gamma y}}{1 + \alpha e^{-\gamma y}} - \frac{\epsilon}{2}(\beta - y) \right] \frac{\partial}{\partial y}. \tag{6.8}$$

In this approximation, $p(t, y, y_0; \beta)$ satisfies

$$\frac{\partial p}{\partial t} = L_y^* p \quad p(0, y_0, y_0; \beta) = \delta(y - y_0), \tag{6.9}$$

where L_y^* is the adjoint of L_y . The function $f(t, t', y_0, \beta)$ satisfies

$$\left. \begin{aligned} \frac{\partial f}{\partial t'} + L_{y_0} f - k_4 e^{-\gamma_2/\beta} e^{-\gamma_1/y_0} f &= 0 \\ f(T, T, y_0, \beta) &= 1 \end{aligned} \right\} \tag{6.10}$$

Once these functions are calculated, it is possible to obtain the open loop controls by nonlinear programming [18]. In other cases, stochastic control theory must be used [19].

7. CONTROL OF FLUCTUATIONS: PARAMETER OPTIMIZATION AND THE ORNSTEIN-UHLENBECK APPROXIMATION

We now consider the simplest possible approximation for controlling the reactor by an open loop control. In this open loop control, the temperature parameter β is fixed at time 0 and kept constant over the entire course of the reaction.

Before discussing the objective functionals, the Ornstein-Uhlenbeck (O-U) approximation is given. The starting point is Eq. (3.9), which is written as

$$dY = f(Y, \beta)dt + (\beta - Y)\sqrt{\epsilon} dW. \tag{7.1}$$

In this equation $dW = \bar{F}_2 dt$ is the increment in Brownian motion [17]. The deterministic steady states, denoted by $y_i(\beta)$, $i = 0, 1, 2$, are solutions of $f(y, \beta) = 0$. We assume $y_0(\beta) \leq y_1(\beta) \leq y_2(\beta)$, with y_0 and y_2 stable and y_1 unstable. Set

$$y^*(\beta) = \frac{1}{2}(y_1(\beta) + y_2(\beta)). \tag{7.2}$$

In the O-U approximation, we assume $Y(0) = y_2$ and replace (7.1) by its Taylor expansion:

$$dY \approx f'(y_2, \beta)(Y - y_2)dt + (\beta - y_2(\beta))\sqrt{\epsilon} dW. \tag{7.3}$$

Now set $X = Y - y_2$, $-\hat{\gamma}(\beta) = f'(y_2, \beta)$, $a(\beta) = \epsilon(\beta - y_2)^2$ to obtain

$$dX = -\hat{\gamma}X dt + \sqrt{a(\beta)} dW \quad X(0) = 0. \tag{7.4}$$

†Since the SDEs studied here are nonlinear, many interpretations are possible. We will use the Stratonovich interpretation [17].

The objective functional used in this problem is the containment probability functional discussed earlier. Set

$$J(T, \beta) = \int_0^T \text{Prob}\{x^*(\beta) \leq X(t) \leq x^+\} dt. \tag{7.5}$$

Here $x^*(\beta) = y^*(\beta) - y_2 = \frac{1}{2}(y_1(\beta) - y_2(\beta))$ and $x^+ = y^+ - y_2$. The X process is an O-U process, and it starts at the x origin; its transition density is [20]

$$\rho(\xi, t, 0) = \left[\frac{\hat{\gamma}}{\pi a(\beta)(1 - e^{-2\hat{\gamma}t})} \right]^{1/2} \exp\left[\frac{-\hat{\gamma}\xi^2}{a(\beta)(1 - e^{-2\hat{\gamma}t})} \right]. \tag{7.6}$$

Let $\sigma^2(t) = (1 - e^{-2\hat{\gamma}(\beta)t})a(\beta)/2\hat{\gamma}(\beta)$. Then

$$\text{Prob}\{x^*(\beta) \leq X(t) \leq x^+\} = \int_{x^*(\beta)}^{x^+} \frac{1}{\sqrt{2\pi\sigma(t)}} \exp\left[\frac{-\xi^2}{2\sigma^2(t)} \right] d\xi. \tag{7.7}$$

Another change of variables gives

$$\text{Prob}\{x^*(\beta) \leq X(t) \leq x^+\} = \Phi(x^+/\sigma(t)) - \Phi(x^*(\beta)/\sigma(t)), \tag{7.8}$$

where

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-x}^z e^{-y^2/2} dy;$$

so that

$$J(T, \beta) = \int_0^T \left[\Phi\left(\frac{x^+}{\sigma(t)}\right) - \Phi\left(\frac{x^*(\beta)}{\sigma(t)}\right) \right] dt. \tag{7.9}$$

The open loop control is picked by finding the value of β that maximizes $J(T, \beta)$.

8. NUMERICAL RESULTS: PARAMETER OPTIMIZATION

In this section, we present some numerical results for the parameter optimization problem, using the methods described in the previous section. The objective functional is $J(T, \beta)$ given by Eq. (7.9) and is a measure of the probability that the system stays in the desired operating region (y^*, y^+) for the entire course of operation. If the system does stay in this region for the entire period, then $J(T, \beta)/T = 1$.

Two values of $y^+ = x^+ + y_2$ were used: $y^+ = 5.1$ ($T^+ = 2040^\circ\text{C}$) and $y^+ = 6$ ($T^+ = 2400^\circ\text{C}$). Three operating periods were considered: $T = 1$, $T = 6$, and $T = 11$. A numerical search for the optimum was done. The smallest increment in β considered was $\Delta\beta = 0.01$ (i.e., 4°C).

When $y^+ = 5.1$, the optimal value of β is $\beta = 1.97$, giving the values $J(1, 1.97) = 0.970419$, $J(6, 1.97) = 5.487472$, $J(11, 1.97) = 9.96478$. When $y^+ = 6.0$, the optimal value of β is $\beta = 1.96$, giving the values $J(1, 1.96) = 0.9900017$, $J(6, 1.96) = 5.926759$, $J(11, 1.96) = 10.859879$. The functional is rather flat near the optimum. For example, for $y^+ = 5.1$, $T = 1$, as β varies from 1.96 to 2.04, the functional $J(1, \beta)$ differs from $J(1, 1.97)$ by a maximum of 3%.

In both cases ($y^+ = 5.1$ or 6), the value of the functional $J(T, \beta)/T$ at the optimal value of β is close to 1 (varying from 0.906 to 0.970 in the case of $y^+ = 5.1$ and from 0.987 to 0.990 in the case of $y^+ = 6$). This indicates that there is a high probability that the system

stays in the desired operating region. This probability decreases as the size of the operating region decreases and decreases as the operating time increases. Other factors that decrease the value of the functional are an increase in the intensity of the noise, a decrease in the size of $\hat{\gamma}$ and moving the initial value of y away from the midpoint of the operating region.

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APPENDIX

Comparison with the model of Luss and co-workers

Ervin and Luss [2] derived a model to describe flickering. The major difference between the model developed here and in [2] is the model of turbulence. In [2], it is assumed that

$$\frac{\tilde{u}}{u_c} = b \sin \omega t, \quad (\text{A-1})$$

where b and w are adjustable parameters. They then assumed that

$$\begin{aligned}k_1(t) &= k_{1c}(1 + b \sin wt)^n \\ h(t) &= h_c(1 + b \sin wt)^n.\end{aligned}\tag{A-2}$$

The analysis of the equations describing flickering thus reduces to studying a deterministic equation with a periodic forcing function.